Leverage and Dividend Irrelevancy Under Corporate and Personal Taxation

HARRY DeANGELO and RONALD W. MASULIS*

1. Introduction and Summary

In "Debt and Taxes," Merton Miller argues that the marginal personal tax disadvantage of debt combined with supply-side adjustments by firms will over-ride the corporate tax advantage of debt and drive market prices to an equilibrium implying leverage irrelevancy to individual firms. Miller's seminal work has significantly enhanced our understanding of the effects of personal taxes on corporate financial decisions and has already stimulated a substantial body of research (e.g., see Kim-Lewellen-McConnell (1979) and Miller-Scholes (1979)). In this paper, we generalize Miller's work in a number of dimensions and conclude that:

1. There are two key properties of the demand-supply interactions of investors and firms which lead to firm level leverage irrelevancy in market equilibrium (section 2).

2. The key demand-side property reveals that the leverage irrelevancy theorem is robust to alternative assumptions about the personal tax code. Moreover, no single security ownership clientele effect is uniquely associated with the theorem. Many different (simple or complex) personal tax codes lead to the theorem and are associated with different (simple and complex) ownership patterns (section 3).

3. In market equilibrium, leverage is irrelevant for firms which issue risky debt even though part of the corporate debt tax shelter is lost in default and re-capture is not allowed (section 4).

4. Even in complete markets, supply-side adjustments by firms which are constrained to issue only conventional securities are not always powerful enough to establish equilibrium prices which imply leverage irrelevancy to individual firms, i.e., leverage relevancy can obtain in equilibrium with complete markets (section 4).

5. When dividend-specific personal tax shelters (e.g., the exclusion) exist,

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equilibrium prices will adjust to imply that any given firm is indifferent among all debt, dividend, and capital gains packages of earnings. Without a dividend-specific personal tax shelter, dividends will not be supplied or demanded in market equilibrium. Nor will dividends be held in market equilibrium when only the Miller-Scholes (1979) borrow-to-shield dividends personal tax shelter is available to individuals (section 5).

2. The Leverage Irrelevancy Theorem Under Uncertainty

We employ a two-date state-preference model in which, at \( t = 0 \), value-maximizing firms sell debt and equity claims against \( t = 1 \) earnings, with debt charges deductible in calculating the corporate tax bill. Utility maximizing investors purchase firms' debt and equity claims and are taxed at personal rates which differ across investors and security classes. Both debt and equity markets are assumed complete, perfectly competitive, and frictionless but are effectively segmented against personal tax arbitrage.

2.1 Supply

For a given firm, define for each state \( s \)

\[
\begin{align*}
X(s) & = \text{earnings before interest and taxes at } t = 1 \\
B(s) & = \text{before personal tax dollars paid to debtholders} \\
E(s) & = (1 - \tau_c)[X(s) - B(s)] = \text{before personal tax dollars paid to equityholders}^1 \\
\tau_c & = \text{cross-sectionally constant corporate tax rate} \\
P_B(s), P_E(s) & = \text{current market prices per state } s \text{ dollar of before personal tax debt and equity income respectively}
\end{align*}
\]

The optimal financing plan maximizes firm value

\[
\text{Max} \quad V = \sum_s P_B(s)B(s) + \sum_s P_E(s)(1 - \tau_c)[X(s) - B(s)] \quad (1)
\]

In (1), the firm has complete flexibility in adjusting its supply of state-contingent debt and equity claims — i.e. \( B(s) \) can be adjusted separately for each state (see section 4). The crucial feature of (1) is that the firm's optimal \( B(s) \) depends only

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1 The assumptions that (i) all corporate debt charges are deductible, (ii) corporations are taxed on their \( t = 1 \) values, and (iii) individuals are taxed on their \( t = 1 \) wealth can be viewed as one-period approximations to a multi-period income tax formulation.

2 The use of the single price law within each market segment does not require firms to issue only simple state claims: it suffices to assume that the positive orthant of statespace is spanned by linear combinations of security payoff vectors for both debt and equity market segments (e.g. see Litzenberger-Van Horne [1977] for use of the same theoretical device for the pricing of conventional debt and equity, not Arrow-Debreu securities).
Leverage and Dividend Irrelevancy

on the sign of the marginal value of state $s$ debt, a market determined constant independent of $B(s)$:

$$\frac{\partial V}{\partial B(s)} = P_B(s) - P_E(s)(1 - \tau_c) \equiv 0$$  \hspace{1cm} (1*)

From (1*), we can characterize the three possible optimal financing solutions to the value-maximization problem (1). If the market offers a state $s$ after-tax debt price premium [$P_B(s) > P_E(s)(1 - \tau_c)$], then (1*) is everywhere positive and the firm will supply only debt claims—i.e., $B(s) = X(s)$ and $E(s) = 0$ is the unique optimal solution to (1). Given an after-tax equity price premium [$P_B(s) < P_E(s)(1 - \tau_c)$], (1*) is everywhere negative and the firm will issue only equity. A noncorner optimum is possible only in the absence of a premium [$P_B(s) = P_E(s)(1 - \tau_c)$] which implies that (1*) is identically zero and the choice of $B(s)$ is irrelevant.

Since all firms face the same market prices and by assumption the same corporate tax rate, the analysis applies to each and every firm. Thus, firms' aggregate supply behavior is characterized by (see Table 1):

**Aggregate Supply Response (ASR):** For all firms and for all levels of leverage, the marginal value of state $s$ debt is the same constant ($= P_B(s) - P_E(s)(1 - \tau_c)$), which depends only on market prices for state $s$ claims and the cross-sectionally constant corporate tax rate. It follows that all firms will exploit debt or equity after corporate tax price premiums ($P_B(s) > \text{ or } < P_E(s)(1 - \tau_c)$) by selling their entire supply of state $s$ claims in the market offering the premium. Only if $P_B(s) = P_E(s)(1 - \tau_c)$ can there simultaneously be positive aggregate supplies of debt and equity claims.

### 2.2 Demand

To see how the differential personal tax treatment of debt and equity income affects aggregate demand for the two classes of securities, consider first the special case of a proportional tax code. Formally, individual $i$’s debt-equity demand depends on personal preferences represented by a utility function $U'[\{Y'(s)\}]$ defined over a vector of after-personal tax state-contingent consumption which is constrained by wealth $W_i$, market prices ($P_B(s), P_E(s)$), and personal tax status denoted by fixed personal tax rates on debt income $\tau_{PB}$ and on equity income $\tau_{PE}$. Individual $i$ selects debt and equity claims to maximize utility subject to a budget constraint, tax status constraints mapping pre-tax into

<table>
<thead>
<tr>
<th>Market Prices</th>
<th>Aggregate supply of state $s$ debt claims</th>
<th>Aggregate supply of state $s$ equity claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_B(s) &lt; P_E(s)(1 - \tau_c)$</td>
<td>zero</td>
<td>positive</td>
</tr>
<tr>
<td>equity price premium</td>
<td>positive</td>
<td>zero</td>
</tr>
<tr>
<td>$P_B(s) &gt; P_E(s)(1 - \tau_c)$</td>
<td>case 1. zero</td>
<td>case 1. positive</td>
</tr>
<tr>
<td>debt price premium</td>
<td>case 2. positive</td>
<td>case 2. zero</td>
</tr>
<tr>
<td>$P_B(s) = P_E(s)(1 - \tau_c)$</td>
<td>case 3. positive</td>
<td>case 3. positive</td>
</tr>
<tr>
<td>no price premium</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
post-tax consumption, and non-negativity constraints eliminating personal tax arbitrage.³

\[
\text{Max}_{B^i(s), E^i(s)} U^i[(Y^i(s))] 
\]

subject to

\[
\sum_s P_B(s)B^i(s) + \sum_s P_E(s)E^i(s) = W^i
\]

\[
Y^i(s) = (1 - \tau^B_{PB})B^i(s) + (1 - \tau^B_{PE})E^i(s); \quad Y^i(s), B^i(s), E^i(s) \geq 0
\]

Under this proportional tax code, utility maximization requires that, for each state s, individual i plunge in the claim (corporate debt or equity) with the higher after-personal tax yield.

To characterize the aggregate demand for debt and equity claims, assume that personal tax rates vary both across debt and equity income and across investors in such a way that at least one individual is in each of the following three (mutually exclusive and exhaustive) marginal personal tax brackets:⁴

Bracket B1: \( (1 - \tau^B_{PB}) > (1 - \tau^B_{PE})(1 - \tau_e) \)

B2: \( (1 - \tau^B_{PB}) = (1 - \tau^B_{PE})(1 - \tau_e) \)

B3: \( (1 - \tau^B_{PB}) < (1 - \tau^B_{PE})(1 - \tau_e) \)

For any given market prices, individuals will adjust their portfolio holdings as shown in Table 2. When equity claims sell at an after-corporate tax premium, as shown in row 1, the after-personal tax yield on debt exceeds that on equity for individuals in brackets B2 and B1 motivating investors in these two brackets to demand positive quantities of debt. When debt sells at an after-corporate tax premium, as shown in row 2, bracket B2 and B3 investors demand equity. When neither debt nor equity sells at an after-corporate tax premium, as shown in row 3, (a condition which is shown below to be necessary for market equilibrium), the after-personal tax yield on debt exceeds, equals, or falls below that on equity for individuals in brackets B1, B2 and B3 respectively, implying a distinct clientele effect: individuals in bracket B1 demand only debt, individuals in bracket B3 demand only equity, and individuals in bracket B2 are the marginal investors in state s securities who are indifferent to buying debt or equity claims because they obtain equal after personal tax yields on debt and equity. Summarizing this individual investor behavior yields the important aggregate demand condition:

³ Personal tax arbitrage can occur, for example, if high tax bracket investors sell heavily taxed personal bonds (or equivalently short sell corporate debt) to low tax bracket investors and purchase lightly taxed corporate equities with the proceeds. The high tax bracket investors gain by obtaining a tax shelter and the low tax bracket investors gain by holding assets with higher before tax yields. The end result is to drive the total personal tax bill to zero. The tax arbitrage constraints \( B^i(s), E^i(s) \geq 0 \) prevent individuals from taking simultaneous short and long positions in state-contingent claims to capture unlimited personal tax subsidies. The constraints are imposed only for simplicity at this stage and are not required to establish irrelevancy—i.e., we can allow short positions as long as the personal tax code builds in other tax arbitrage safeguards (e.g. see section 5).

⁴ For example, in Miller’s special case in which equity income is not taxed (\( \tau_{PE} = 0 \)), B1 investors have relatively low personal tax rates on debt income (\( \tau_{PB} < \tau_e \)), B2 investors have intermediate tax rates (\( \tau_{PB} = \tau_e \)), and B3 investors have relatively high tax rates (\( \tau_{PB} > \tau_e \)).
Table 2

(Proportional Tax Code)

Individuals' Aggregate Debt-Equity Demand Decision Induced by Personal Tax Rates and Market Prices

<table>
<thead>
<tr>
<th>Market Prices</th>
<th>After-personal Tax Yields on State s Debt and Equity</th>
<th>Aggregate Demand for State s Debt and Equity by Tax Brackets</th>
<th>Tax Bracket of Marginal Investors and the Implied Marginal Tax Rate Condition*</th>
</tr>
</thead>
</table>
| $P_d(s) < P_e(s) (1 - \tau_c)$         | $1 - \tau_d P_d(s) > 1 - \tau_e P_e(s)$               | Positive demand for debt                                     | Bracket B3
| equity price premium (after corporate tax) |                                                       |                                                             | $(1 - \tau_d) < (1 - \tau_e) (1 - \tau_c)$                                    |
|                                        | $1 - \tau_d P_d(s) \geq 1 - \tau_e P_e(s)$             | Positive demand for equity                                   |                                                                            |
|                                        |                                                         | Positive demand for equity or debt or indifference            |                                                                            |
| $P_d(s) > P_e(s) (1 - \tau_c)$         | $1 - \tau_d P_d(s) < 1 - \tau_e P_e(s)$               | Positive demand for equity                                   | Bracket B1
| debt price premium (after corporate tax) |                                                       |                                                             | $(1 - \tau_d) > (1 - \tau_e) (1 - \tau_c)$                                    |
|                                        | $1 - \tau_d P_d(s) \leq 1 - \tau_e P_e(s)$             | Positive demand for debt or equity or indifference            |                                                                            |
| $P_d(s) = P_e(s) (1 - \tau_c)$         | $1 - \tau_d P_d(s) = 1 - \tau_e P_e(s)$               | Positive demand for debt                                     | Bracket B2
| no price premium (after corporate tax) |                                                       |                                                             | $(1 - \tau_d) = (1 - \tau_e) (1 - \tau_c)$                                    |
|                                        | $1 - \tau_d P_d(s) = 1 - \tau_e P_e(s)$               | Indifference between debt and equity                         |                                                                            |
|                                        | $1 - \tau_d P_d(s) < 1 - \tau_e P_e(s)$               | Positive demand for equity                                   |                                                                            |

* (denote $i = \mu$ for the marginal investors who are indifferent between debt and equity because after personal tax yields on debt and equity are equated at given market prices)

Note that $\frac{1}{P(s)}$ is the before tax yield, since $P(s)$ is the current price of a claim to pay $1$ in state $s$. 
Tax-Induced Positive Aggregate Demand (TIPAD): The personal tax treatment of debt and equity income is sufficiently heterogeneous that at least one individual demands the debt or equity claim priced at a discount (i.e., state $s$ debt when $P_B(s) < P_E(s)(1 - \tau_c)$ or equity when $P_B(s) > P_E(s)(1 - \tau_c)$). If $P_B(s) = P_E(s)(1 - \tau_c)$, there is a positive aggregate demand for both claims.

2.3 Market Equilibrium

Combining ASR and TIPAD (or Tables 1 and 2), we see that market equilibrium requires $P_B(s) = P_E(s)(1 - \tau_c)$ for all $s$ since only then can the desires of individuals and firms be simultaneously satisfied. On the supply side, all firms obtain the same constant marginal value of debt for all levels of leverage so that all respond identically by supplying only the debt or equity claim priced at a premium. Debt and equity can be in positive aggregate supply simultaneously only in the absence of a price premium, which implies that each firm is indifferent to leverage. On the demand side, the heterogeneous personal tax treatment of different investors' debt and equity income ensures that some investors will demand the debt or equity claim priced at a discount. Given no price premium, there will be positive aggregate demand for both debt and equity claims. Together ASR and TIPAD preclude debt or equity from being a totally dominant form of financing and yield the equilibrium pricing condition $P_B(s) = P_E(s)(1 - \tau_c)$ [or the equivalent marginal tax rate condition $(1 - \tau_B) = (1 - \tau_E)(1 - \tau_c)$] which implies leverage irrelevancy for the individual firm.\(^5\) The specifics of the equilibrium pattern of securities ownership will depend on the particular features found in the personal tax code (see section 3). In sum, we obtain

**Leverage Irrelevancy Theorem:** Given ASR and TIPAD, market equilibrium implies (i) the leverage decision (debt-equity packaging of earnings) is irrelevant to the valuation of any given firm and (ii) the aggregate supplies of corporate debt and equity are socially relevant in the sense that in the aggregate, investors demand positive quantities of debt and equity claims in order to arrange their portfolios in the most tax efficient manner.

3. The Personal Tax Code, Clientele Effects, and Leverage Irrelevancy

The leverage irrelevancy theorem is robust to alternative specifications of the personal tax code and is not associated with any one particular clientele effect in security ownership. Any personal tax code which implies TIPAD will, when combined with ASR, lead to the theorem. Many different (simple or complex) personal tax codes imply TIPAD and are associated with different (simple or complex) ownership patterns. For example, marginal personal tax rates which are

\(^5\) In our formulation (1), the firm selects a financing plan to maximize its net market value. Equivalently, the firm may be viewed as selecting a financing plan to maximize the after-personal tax cash flows to marginal security holders in the market place:

$$\max_{B(s)} \text{ATCF} = \sum_s (1 - \tau^B_B)B(s) + \sum_s (1 - \tau^E_E)(1 - \tau_c)(X(s) - B(s)).$$

The after-tax cash flow maximization condition is readily shown to be equivalent to (1*).
increasing or decreasing functions of income (but which treat debt and equity income sufficiently differently) are readily shown to imply TIPAD and more complex ownership patterns. Also, adding tax-exempt municipals (or any securities with personal tax status different from corporate debt and equity) does not affect the theorem provided that debt and equity remain sufficiently attractive to some investors so that TIPAD holds. However, predicted clientele ownership patterns would be more complex if municipals or other securities were available.⁶

It follows that a test of Miller’s model based on predicted clientele effects, for example, Kim, Lewellen and McConnell (1979), is properly viewed as a joint test of the leverage irrelevancy theorem and the particular personal tax code which implies TIPAD. Thus, an observed ownership pattern which is inconsistent with the predicted clientele pattern may be due to a misspecification of the personal tax code rather than to a failure of the leverage irrelevancy theorem. Perhaps a more important limitation of clientele effect tests of leverage irrelevancy is that they are incapable of distinguishing between a world in which leverage is irrelevant and a world in which the same security ownership patterns obtain but leverage matters to individual firms (e.g., as can be true with corporate tax shelter substitutes for debt such as accounting depreciation—see DeAngelo-Masulis (1980)).

4. Conventional Debt, Default Risk, and Supply-Side Flexibility

Our proof of the leverage irrelevancy theorem allows each firm total supply flexibility; it can choose state by state whether to label any part (or all) of its earnings as debt or equity. In other words, firms are allowed to issue conventional debt and residual equity securities (bundles of state-contingent claims), but they are not constrained to do so.

If the market equilibrium price structure of Section 2 holds, leverage is irrelevant for firms issuing conventional debt and equity securities. Moreover this irrelevancy result obtains over the risk-free debt range (as considered by Miller) as well as over higher debt levels which involve potential default. To see this, let \( B \) denote the promised payment to debt in all states (conventional debt decision variable) so that \( B(s) = \min[B, X(s)] \) and \( E(s) = (1 - \tau_c)\max[X(s) - B, 0] \). Firm value is then

\[
V = \sum_s P_B(s)\min[B, X(s)] + \sum_s P_E(s)(1 - \tau_c)\max[X(s) - B, 0]
\]

The marginal value of debt-financing is now

\[
\frac{\partial V}{\partial B} = \sum_{s \in \Omega} [P_B(s) - P_E(s)(1 - \tau_c)]
\]

where \( \Omega \) is the no-default states \( = \{ s: X(s) \geq B \} \). Now, if \( P_B(s) = P_E(s)(1 - \tau_c) \)

⁶ Also, many realistic personal tax shelters (e.g. individual-specific deductions and credits) can be shown to be consistent with TIPAD. Moreover, special provisions or “loop-holes” in the personal tax code can actually create the heterogeneous treatment of income from various classes of securities which leads to TIPAD and irrelevancy. For example, the personal dividend exclusion implies that a positive quantity of dividend paying equities would be demanded (and supplied) for their tax-avoidance attributes and dividend policy would be irrelevant to the individual firm (see section 5).
for all \( s \), then \( \partial V / \partial B = 0 \) for all \( B \) and leverage is indeed irrelevant to the firm even with a positive default probability. Notice that irrelevancy obtains with risky debt even though part of the corporate debt tax shelter is lost in default and individuals do not recapture this lost shelter at the personal level. The intuitive explanation is that any corporate debt shelter lost in default is associated with an equal reduction in the personal tax liabilities of the marginal investors in equilibrium due to the reduction in debt income received.\(^7\)

A more difficult question is whether \( P_B(s) = P_E(s)(1 - \tau_c) \) characterizes market equilibrium in our complete markets economy if firms are constrained to issue only conventional securities as assumed by Miller [1977] and Kim-Lewellen-McConnell [1979]. Unfortunately, this price relationship will not always hold in equilibrium. When constrained to conventional securities, firms' supply responses cannot eliminate price premiums in all cases, and therefore the leverage irrelevancy conclusion can also fail. Conventional securities have inherent supply inflexibilities insofar as only particular bundlings of state claims are feasible for a given firm. Specifically, conventional securities effectively force a firm to jointly supply the same total dollar payment to debtholders in all non-default states and to supply a payment to debtholders equal to total earnings in all default states. As a result, each firm may be unable to adjust its supply of conventional debt and equity securities to capture an after-tax price premium in a given state without incurring even greater price discounts in other states. More concretely, leverage can matter in equilibrium because conventional securities limit firms' abilities (not incentives) to expand their supply of, say, state 1 debt claims at prices \( P_B(1) > P_E(1)(1 - \tau_c) \) because to do so would also require a disadvantageous state 2 debt supply expansion at prices \( P_B(2) < P_E(2)(1 - \tau_c) \).

Nevertheless, it is possible for the leverage irrelevancy theorem to hold when firms are constrained to issue conventional securities. The theorem would con-

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7 Allowing recapture of corporate tax shelter will invalidate the irrelevancy theorem if recapture in default generates additional corporate tax benefits without recognition of offsetting additional personal tax liabilities.

8 An example should help clarify how constraining firms to conventional securities can destroy the irrelevancy theorem. Assume there are two possible states (\( s = 1, 2 \)) and, if necessary, reorder the states so that \( 0 < X(1) < X(2) \) for a given firm. Define the price premium function \( P(s) = P_B(s) - P_E(s) \). Then, from (2), the marginal value of debt financing is

\[
\frac{\partial V}{\partial B} = P(1) + P(2) \quad \text{for} \quad 0 \leq B \leq X(1)
\]

\[
= P(2) \quad \text{for} \quad X(1) \leq B \leq X(2)
\]

For this example a possible market equilibrium price structure is one for which \( P(s) \neq 0 \)—i.e., we can have \( P(1) > 0, P(2) < 0, \) and \( P(1) + P(2) < 0 \) in equilibrium. In this case, leverage matters and \( B = 0 \) is the unique optimum capital structure for this firm. With only conventional securities available, the firm is unable to adjust leverage to capture the debt price premium in state 1 without simultaneously sacrificing the even larger discount in state 2. Moreover, both debt and equity markets are easily shown to be complete so that supply can equal demand in each market at current prices.

This example demonstrates that, even though both debt and equity markets are complete, firms may be unable to compete away price premiums by adjusting supplies of conventional securities. Of course, by their very nature, incomplete markets (constraints on the economy's risk-sharing capabilities) presuppose constraints on firms' supply-adjustment capabilities and, a fortiori, we would not expect irrelevancy to obtain in general.
tinue to hold if sufficient firms could profitably respond to, say, a state's debt price premium by repackaging state's equity claims into state's debt claims so that markets no longer cleared at the existing price premium. In other words, constraints on the economy's aggregate supply response capability need not be binding at equilibrium. Moreover, there are market forces acting to break down any such constraint. The existence of an after-corporate tax premium for a debt or equity claim in a given state provides firms with the incentive (i) to alter their state contingent earnings distributions and/or (ii) to design special contracts which enable them to supply additional claims in the market offering the premium and, in so doing, compete it away. Alternatively, assumptions about the demand structure (e.g., economy-wide risk-neutrality with homogeneous beliefs, see DeAngelo-Masulis (1980)) can render conventional-security supply constraints nonbinding.

Since the leverage irrelevancy theorem will not necessarily fail if firms can issue only conventional securities, the more realistic assumption of conventional securities can validly be incorporated in Miller's supply-adjustment model. However, one should recognize that conventional security supply adjustments alone may not compete away all premiums in market equilibrium and other assumptions may be required before supply adjustments are sufficiently powerful to determine relative prices.

5. Dividend Irrelevancy

We can extend our formulation to include the firm's choice of cash dividend and capital gain composition of equity financing. We find that when dividend-specific personal tax shelters (e.g., the personal dividend exclusion) are introduced, equilibrium relative prices will adjust so that any given firm is indifferent among all possible debt, dividend, and capital gain packageings of earnings. However, without a dividend-specific tax shelter, no dividends will be paid in market equilibrium. Nor will dividends be paid in market equilibrium in the Miller-Scholes (1979) case in which a personal tax deduction for borrowing is available to investors but there is no dividend-specific personal tax shelter such as the dividend exclusion.

To capture the differential personal tax treatment of cash dividends and long term capital gains, separate state's equity claims into respective dividend and capital gain components: 

\[ E(s) = (1 - \tau_c)[X(s) - B(s)] = D(s) + G(s) \]

The value of the firm is then

\[ V = \sum_s P_B(s)B(s) + \sum_s P_D(s)D(s) + \sum_s P_G(s)[(1 - \tau_c)[X(s) - B(s)] - D(s)] \] 

(3)

where \( P_B(s), P_D(s), \) and \( P_G(s) \) are the current prices of state's before personal tax debt, dividends, and gains.

Examining (3) and applying the logic of section 2.1, we have

**Generalized Aggregate Supply Response (ASR):** Every firm will supply only
the security with the highest after-corporate tax price. Consequently, positive quantities of state corporate debt, dividends, and gains can be supplied in the aggregate simultaneously only if there is no after-corporate tax price premium.

\[ P_B(s) = P_D(s)(1 - \tau_c) = P_G(s)(1 - \tau_c) \]  

(4)

Of course, (4) implies that the choice among debt, dividends, and gains is a matter of indifference to any given firm.

On the personal level, let \( \tau_{P-I} \) and \( \tau_{P-G} \) denote individual \( i \)'s tax rates on ordinary income and capital gains respectively with \( \tau_{P-I} \geq \tau_{P-G} \geq 0 \) where we assume that capital gains are taxed as accrued. Let \( \epsilon \geq 0 \) represent each individual's personal dividend exclusion. We initially rule out personal borrowing \( (B^i(s) < 0) \) but later relax this condition. Paralleling section 2.2's development, individual \( i \)'s choice problem is:

\[
\max_{D^i(s), G^i(s), B^i(s)} U^i(\{Y^i(s)\})
\]

subject to

\[
\sum_s P_B(s)B^i(s) + \sum_s P_D(s)D^i(s) + \sum_s P_G(s)G^i(s) = W^i
\]

\[
Y^i(s) = G^i(s)(1 - \tau_{P-G}) + B^i(s) + D^i(s) - \tau_{P-I} \max\{B^i(s) + \max\{D^i(s) - \epsilon, 0\}, 0\}
\]

Consider first the case in which no dividend-specific tax shelter exists \( (\epsilon = 0) \). Then, as in section 2.2, utility maximization requires investors to plunge in the claim with the highest after personal tax yield. Given the pricing relationship (4) so that positive supplies of all three claims are possible, investor \( i \)'s after personal tax yields on gains, dividends, and debt respectively are given by

\[
1 - \frac{\tau_{P-G}}{P_G(s)} < 1 - \frac{\tau_{P-I}}{P_D(s)} < 1 - \frac{\tau_{P-I}}{P_B(s)}
\]

Consequently, no rational investor will choose to hold dividends at these prices as debt strictly dominates dividends. Given (4), after personal tax yields dictate that investors with tax rates \( (1 - \tau_{P-I}) > (1 - \tau_{P-G})(1 - \tau_c) \) will hold only debt while investors with \( (1 - \tau_{P-I}) < (1 - \tau_{P-G})(1 - \tau_c) \) will strictly prefer gains. Investors with \( (1 - \tau_{P-I}) = (1 - \tau_{P-G})(1 - \tau_c) \) are the marginal investors who are indifferent between debt and gains. Thus, without a personal dividend exclusion a generalized tax induced positive aggregate demand (TIPAD) condition holds for debt and capital gains but not for dividends. Given the generalized ASR condition, it follows that in market equilibrium, relative prices will satisfy \( P_B(s) \)

\[ \text{There will be a zero aggregate supply of state corporate dividends if relative prices dictate a firm level advantage to gains over dividends} \ (P_G(s) > P_D(s)) \text{ and/or a firm level advantage to debt over dividends} \ (P_B(s) > P_D(s)(1 - \tau_c)). \text{ Similarly, there will be a zero aggregate supply of state gains if} \ P_D(s) > P_G(s) \ 	ext{and/or} \ P_B(s) > P_G(s)(1 - \tau_c). \text{ And there will be a zero aggregate supply of state debt if} \ P_B(s) < P_D(s)(1 - \tau_c) \ 	ext{and/or} \ P_B(s) < P_G(s)(1 - \tau_c). \]
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This zero-dividend equilibrium will still obtain, if as in Miller-Scholes (1979), individuals can borrow (set \( B'(s) < 0 \)) as well as lend and utilize personal debt charges as a deduction to offset dividend income (\( D'(s) > 0 \)). To introduce the Miller-Scholes personal debt tax shelter in our complete markets model, remove the no personal borrowing constraint \( B'(s) \geq 0 \) constraint in (4). With \( \epsilon = 0 \), the ordinary income tax bill is \( \tau_{IV} \max[B'(s) + D'(s), 0] \) where the deduction for personal net borrowing (\( B'(s) < 0 \)) is limited to the amount of investment income received (\( D'(s) \geq 0 \)). This formalization of the Miller-Scholes shelter assumes that the market treats personal debt as a perfect substitute for corporate debt with unit price \( PB(S) \).

Without affecting any of our conclusions, we could allow tax-free accumulation of gains by setting \( TPG = 0 \) for all investors. In this case, individual holdings of equity gains are perfect substitutes for tax-free municipal bonds or insurance policies which offer an equal yield of \( 1/PG(S) \). Our conclusions also remain unchanged if we set \( \tau_{IV} = \tau_{IVG} = 0 \) for some individuals to allow for the existence of some tax-exempt investors.

The zero dividends equilibrium continues to obtain under these additional assumptions because (i) relative prices \( PB(s) \leq PD(s)(1 - \tau_c) < PD(s) \) which are necessary but not sufficient to induce firms to supply positive quantities of dividends imply that investors demand zero dividends even if they can borrow to shield dividends from personal taxation and (ii) relative prices \( PB(s) \geq PD(s) \) which are necessary but not sufficient to induce investors to hold dividends (with or without a personal borrowing deduction) will induce firms to supply zero dividends. Since (i) and (ii) encompass all possible relative prices, market equilibrium still requires that zero dividends be demanded and supplied.

To see (i), recall from ASR that a positive aggregate supply of dividends requires \( PB(s) \leq PD(s)(1 - \tau_c) < PD(s) \). However, as before, rational investors will not demand dividends at these prices because the after-personal tax yield on debt is higher. A fortiori, no investor will borrow (i.e., sell personal bonds) to buy an equal quantity of dividend income at prices \( PB(s) < PD(s) \). That is, no rational investor will buy a dividend claim for \( PD(s) \) and then sell that claim for a lower unit price \( PB(s) \). Regarding (ii), similar logic implies that investors would borrow to shield positive holdings of dividend income only if the market’s before personal tax yield on dividends is greater than or equal to the yield on debt—i.e., only if \( PB(s) \geq PD(s) \). But no dividends will be supplied by any firms at these prices. In other words, the relative price conditions which make the Miller-Scholes dividend

A necessary condition for a positive aggregate demand for dividends is that \( PB(s) \geq PD(s) \) since only then will the after personal tax yield on dividends be greater than or equal to that on debt. (Litzenberger-Van Horne (1978, fn 10) make this same point.) But, from ASR, \( PB(s) \geq PD(s) \) implies a zero aggregate supply of dividends. Thus, market equilibrium requires a zero demand and supply of dividends. At equilibrium, relative prices satisfy \( PB(s) = PD(s) \) where \( PD(s) \) must be interpreted as the shadow price of dividends because no dividends are supplied or demanded. At these prices, all firms strictly prefer debt to dividends. Investors whose personal tax situations dictate holding ordinary income are indifferent between holding debt (at price \( PB(s) \)) and dividends (at the perceived shadow price \( PD(s) = PD(s) \)). The supply decisions of firms dictate the market corner solution in which all such investors obtain ordinary income via debt.
shelter attractive to investors will, in our supply adjustment model, induce firms to eliminate all dividends and obviate the usefulness of the shelter. Together, (i) and (ii) imply that when firms are allowed to adjust their supplies in response to market prices, the Miller-Scholes tax shelter does not provide the incentive for positive dividends in market equilibrium.

The introduction of a dividend-specific personal tax shelter \((\epsilon > 0)\) will, in market equilibrium, cause prices to satisfy the indifference condition (4). In this case, there will be positive aggregate supplies of debt, dividends, and capital gains, and each firm will be indifferent among all possible debt, dividends, and capital gains packages of earnings. To establish this generalized financial structure irrelevancy result, notice that given the pricing relationship (4), positive quantities of dividends will be demanded by investors facing tax rates \(\tau_{PI} > \tau_c\) and \(\tau_{PG} > 0\) because over the dividend range \(0 \leq D'(s) \leq \epsilon\), the after-personal tax yield on dividends exceeds that on gains and debt:

\[
\frac{1 - \tau_{PG}}{P_G(s)} < \frac{1}{P_D(s)} > \frac{1 - \tau_{PI}}{P_B(s)}
\]

By a similar comparison of after-tax yields, (4) implies that positive quantities of debt will be demanded by investors with \((1 - \tau_{PI}) > (1 - \tau_{PG})(1 - \tau_c)\) who desire more state s income that \(\epsilon\). Similarly, positive quantities of capital gains will be demanded by investors with \((1 - \tau_{PI}) < (1 - \tau_{PG})(1 - \tau_c)\) who also desire after-tax state s income greater than \(\epsilon\). In sum, positive quantities of all three types of claims are demanded in the aggregate given (4). One can readily show that, with this personal tax code \((\epsilon > 0)\), a violation of (4) will be associated with a positive aggregate demand for the type of claim (or claims) not supplied to the market. In other words, with a dividend-specific personal tax shelter, a generalized TIPAD condition holds for debt, dividends, and gains. Thus, combining ASR and TIPAD exactly as in section 2, equilibrium requires \(P_B(s) = P_D(s)(1 - \tau_c) = P_G(s)(1 - \tau_c)\) which implies that the choice among debt, dividends, and gains is a matter of indifference to any given firm.\(^{11}\)

REFERENCES


\(^{11}\) In equilibrium, no investor will hold dividends in excess of the dividend-specific shelter. Unless dividend-specific shelters other than the exclusion can be identified, our model does not explain the magnitude of observed dividend holdings.